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GENERALISED USEFUL FUZZY INACCURACY MEASURES AND THEIR BOUNDS

Ashiq H. Bhat , M. A. K. Baig
P.G. Department of Statistics
University of Kashmir, Srinagar

***ABSTRACT*:** In this paper we present a new class of generalized useful fuzzy inaccuracy measure. This measure is not only new but some known measures are the particular cases of our proposed measure. We also obtained the bounds for this measure.
***Keywords*:** Fuzzy set, Membership function, Kraft inequality, Coding theorem, Fuzzy entropy, Holders inequality

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1. INTRODUCTION

The concept of fuzzy sets was introduced by Lotfi. A. Zadeh [1] and developed his own theory to measure the ambiguity of a fuzzy set. A fuzzy set "A"is represented as:

$$A= \{x_i /\mu_A(x_i): i=1,2,3,...,n\}$$

Where $\mu_A(x_i)$ gives the degree of belongingness of the element x_i to the set "A" and is defined as follows:

$$\mu_A(x_i) = \begin{cases} 0, \text{if} x_i \notin \text{Aandthereisnoambiguity}, \\ 1, \text{if} x_i \in \text{Andthereisnoambiguity}, \\ 0.5, \text{if} x_i \in \text{Aor} x_i \notin \text{Aandthereismaximumambiguity}, \end{cases}$$

If $x_1,x_2,\ldots,x_n$ are members of the universe of discourse, with respective membership functions $\mu_A(x_1)$, $\mu_A(x_2)$, $\mu_A(x_3)$,…, $\mu_A(x_n)$, then all $\mu_A(x_1), \mu_A(x_2), \mu_A(x_3), \ldots, \mu_A(x_n)$ lies between 0 and 1 but these are not probabilities because their sum is not unity. $\mu_A(x_i)$gives the element$x_i$the degree of belongingness to the set "A".The function $\mu_A(x_i)$associates with each $x_i \in R^n$ a grade of membership to the set "A" and is known as membership function.

The different elements $x_i$depends upon the experimenters goal or upon some qualitative characteristics of the physical system taken into account; ascribe to each element x_i a non-negative number (u_i>0) directly proportional to its importance and call$u_i$the utility of the element x_i.Then the weighted fuzzy entropy [2] of the fuzzy set "A" is defined as:

$$H(A, U) = -\sum_{i=1}^{n} u_i\{\mu_A(x_i) \log \mu_A(x_i) + (1-\mu_A(x_i)) \log(1-\mu_A(x_i))\} \quad (1.1)$$

Now let us suppose that the experimenter asserts that the membership function of the ith element is $\mu_B(x_i)$, where the true membership function is$\mu_A(x_i)$, thus we have two utility fuzzy information schemes:

$$F.S = \begin{bmatrix} x_1 & x_2 & \ldots & x_n \\ \mu_A(x_1) & \mu_A(x_2) & \ldots & \mu_A(x_n) \\ u_1 & u_2 & \ldots & u_n \end{bmatrix}, 0 \le \mu_A(x_i) \le 1 \ \forall x_i, u_i > 0 \quad (1.2)$$

Of a set of n elements after an experiment, and

$$F.S^* = \begin{bmatrix} x_1 & x_2 & \ldots & x_n \\ \mu_B(x_1) & \mu_B(x_2) & \ldots & \mu_B(x_n) \\ u_1 & u_2 & \ldots & u_n \end{bmatrix}, 0 \le \mu_B(x_i) \le 1 \ \forall x_i, u_i > 0 \quad (1.3)$$

of the same set of n elements before the experiment.

In both the schemes (1.1) and (1.2) the utility distribution is the same because we assume that the utility u_i of an element x_i is independent of its membership function $\mu_A(x_i)$, or predicted membership function $\mu_B(x_i)$, u_i is only a 'utility' or value of the element x_i for an observer relative to some specified goal (refer to [3]).

The quantitative-qualitative measure of fuzzy inaccuracy corresponding to Taneja and Tuteja measure of inaccuracy [4] with the above schemes is:

$$I(A;B;U) = -\sum_{i=1}^{n} u_i\{\mu_A(x_i)\log\mu_B(x_i) + (1-\mu_A(x_i))\log(1-\mu_B(x_i))\} \tag{1.4}$$

Guiasu and Picard [5] considered the problem of encoding the letter output by the source (1.2) by means of a single prefix code with code-words $c_1, c_2, \dots, c_n$ having lengths $l_1, l_2, \dots, l_n$ satisfying Kraft [6] inequality:

$$\sum_{i=1}^{n} D^{-l_i} \le 1 \tag{1.5}$$

Where D being the size of the code alphabet, corresponding to Guiasu and Picard [5] useful mean codeword length we have the following useful fuzzy mean length of the code

$$L(A;U) = \frac{\sum_{i=1}^{n} u_i\{\mu_A(x_i) + (1-\mu_A(x_i))\}l_i}{\sum_{i=1}^{n} u_i\{\mu_A(x_i) + (1-\mu_A(x_i))\}} \tag{1.6}$$

and obtain bounds for it in terms of (1.4) under the condition:

$$\sum_{i=1}^{n}\{\mu_A(x_i)\mu_B^{-1}(x_i) + (1-\mu_A(x_i))(1-\mu_B(x_i))^{-1}\} D^{l_i} \le 1 \tag{1.7}$$

Where D is the size of code alphabet, inequality (1.7) is generalized fuzzy Kraft's inequality. A code satisfying generalized fuzzy Kraft's inequality is known as a personal fuzzy code.It is easy to see that for $\mu_A(x_i) = \mu_B(x_i) \forall x_i, i = 1,2,3,\dots,n$ (1.7) reduces to Kraft [6] inequality.

In this paper generalized useful fuzzy code-word mean length are considered and bounds have been obtained in terms of generalized useful fuzzy inaccuracy measure of order α and type β .The main aim of these results is that it generalizes some well-known fuzzy measures already existing in the literature.

2. Generalized measures of useful fuzzy information and their bounds:

Consider a function:

$$I_\alpha^\beta(A;B;U) = \frac{1}{D^{\frac{\alpha-1}{\alpha}}}\left[1 - \left(\frac{\sum_{i=1}^{n} u_i\{\mu_A^\beta(\mathbf{x_i})\mu_B^{\alpha-1}(\mathbf{x_i}) + (1-\mu_A(\mathbf{x_i}))^\beta(1-\mu_B(\mathbf{x_i}))^{\alpha-1}\}}{\sum_{i=1}^{n} u_i\{\mu_A^\beta(x_i) + (1-\mu_A(x_i))^\beta\}}\right)^{\frac{1}{\alpha}}\right] \tag{2.1}$$

Where $\alpha > 0 (\ne 1), \beta > 0, \mu_A(x_i) \ge 0, \mu_B(x_i) \ge 0 \ \forall x_i, i = 1,2,3,\dots,n$. D is the size of the code alphabet.

Remarks for (2.1):

(i) When $\alpha \to 1, \beta = 1$, (2.1) reduces to a measure of useful fuzzy inaccuracy corresponding to Taneja and Tuteja [4] measure of useful inaccuracy.

(ii) When $\beta = 1, \mu_A(x_i) = \mu_B(x_i) \ \forall x_i, i = 1,2,3,\dots,n$, (2.1) reduces to useful fuzzy information measure corresponding to Autar and Khan [7] useful information measure.

(iii) When $\alpha \to 1, \beta = 1$, and the utility of the scheme is ignored, the measure (2.1) becomes the fuzzy inaccuracy measure corresponding to Kerridge [8] measure of inaccuracy.

(iv) When $\alpha \to 1, \beta = 1$, and $\mu_A(x_i) = \mu_B(x_i) \ \forall x_i, i = 1,2,3,\dots,n$ and utility of the scheme is ignored, the measure (2.1) reduces to fuzzy information measure due to De Luca and Termini [9].We call (2.1) as generalized useful fuzzy inaccuracy measure of order α and type β.

Further, consider a generalized useful fuzzy code-word mean length credited with utilities and membership function as:

$$L_\alpha^\beta(A;U) = \frac{1}{D^{\frac{\alpha-1}{\alpha}}}\left[1 - \sum_{i=1}^{n}\{\mu_A^\beta(x_i) + (1-\mu_A(x_i))^\beta\}\left(\frac{u_i}{\sum_{i=1}^{n} u_i\{\mu_A^\beta(x_i) + (1-\mu_A(x_i))^\beta\}}\right)^{\frac{1}{\alpha}} D^{-l_i\left(\frac{\alpha-1}{\alpha}\right)}\right] \tag{2.2}$$

Where $\alpha > 0 (\ne 1), \beta > 0, \mu_A(x_i) \ge 0 \ \forall x_i, i = 1,2,3,\dots,n$. D is the size of the code alphabet.

Remarks for (2.2):

(i) When $\alpha \to 1, \beta = 1$, (2.2) reduces to useful fuzzy mean length of the code corresponding to Guiasu and Picard [5].

(ii) When $\alpha \to 1, \beta = 1$, and the utility of the scheme is ignored, the mean length of the code (2.2) becomes optimal fuzzy code length corresponding to Shannon [10].

Now we find the bounds for (2.2) in terms of (2.1) under the condition:

$$\sum_{i=1}^{n}\{\mu_A^{\beta}(x_i)\,\mu_B^{-1}(x_i) + (1-\mu_A(x_i))^{\beta}(1-\mu_B(x_i))^{-1}\}D^{-l_i} \le 1 \tag{2.3}$$

Where D is the size of the code alphabet.It is easy to see that for $\beta = 1$, and $\mu_A(x_i) = \mu_B(x_i)\ \forall x_i, i = 1,2,3,\dots,n$ inequality (2.3) Kraft [6].

Theorem 2.1:- For all integers D (D> 1). Let l_i satisfies the condition (2.3), then the generalized useful fuzzy code-word mean length satisfies

$$L_\alpha^\beta\ (A;U) \ge I_\alpha^\beta\ (A;B;U) \tag{2.4}$$

Equality holds iff

$$l_i = -\log\left(\frac{u_i(\mu_B^{\alpha}(x_i)+(1-\mu_B(x_i))^{\alpha})}{\sum_{i=1}^{n} u_i\{\mu_A^{\beta}(\mathbf{x_i})\mu_B^{\alpha-1}(\mathbf{x_i})+(1-\mu_A(\mathbf{x_i}))^{\beta}(1-\mu_B(\mathbf{x_i}))^{\alpha-1}\}}\right) \tag{2.5}$$

Proof:-By Holder's inequality we have

$$\sum_{i=1}^{n} x_i y_i \ge \left(\sum_{i=0}^{n} x^p\right)^{\frac{1}{p}}\left(\sum_{i=0}^{n} x^q\right)^{\frac{1}{q}} \tag{2.6}$$

For all $x_i, y_i > 0, i = 1,2,3,\dots,n$ and $\frac{1}{p}+\frac{1}{q} = 1, p < 1(\neq 0), q < 0$ or $q < 1(\neq 0), p < 0$. We see the equality holds iff there exists a positive constant c such that

$$x_i^p = c y_i^q \tag{2.7}$$

Making the substitution

$$x_i = \mu_A^{\frac{\alpha\beta}{\alpha-1}}(x_i) + (1-\mu_A(x_i))^{\frac{\alpha\beta}{\alpha-1}}\left(\frac{u_i}{\sum_{i=1}^{n} u_i\{\mu_A^{\beta}(x_i)+(1-\mu_A(x_i))^{\beta}\}}\right)^{\frac{1}{\alpha-1}} D^{-l_i}$$

$$y_i = \mu_A^{\frac{\beta}{1-\alpha}}(x_i)\mu_B^{-1}(x_i) + (1-\mu_A(x_i))^{\frac{\beta}{1-\alpha}}(1-\mu_B(x_i))^{-1}\left(\frac{u_i}{\sum_{i=1}^{n} u_i\{\mu_A^{\beta}(x_i)+(1-\mu_A(x_i))^{\beta}\}}\right)^{\frac{1}{1-\alpha}}$$

$$p = \frac{\alpha-1}{\alpha}, q = 1-\alpha$$

In (2.6), we get

$$\sum_{i=1}^{n}\{\mu_A^{\beta}(x_i)\,\mu_B^{-1}(x_i) + (1-\mu_A(x_i))^{\beta}(1-\mu_B(x_i))^{-1}\}D^{-l_i} \ge$$

$$\left[\sum_{i=1}^{n}\{\mu_A^{\beta}(x_i) + (1-\mu_A(x_i))^{\beta}\}\left(\frac{u_i}{\sum_{i=1}^{n} u_i\{\mu_A^{\beta}(x_i)+(1-\mu_A(x_i))^{\beta}\}}\right)^{\frac{1}{\alpha}} D^{-l_i\left(\frac{\alpha-1}{\alpha}\right)}\right]^{\frac{\alpha}{\alpha-1}}$$

$$\left[\sum_{i=1}^{n}\{\mu_A^{\beta}(x_i)\,\mu_B^{\alpha-1}(x_i) + (1-\mu_A(x_i))^{\beta}(1-\mu_B(x_i))^{\alpha-1}\}\left(\frac{u_i}{\sum_{i=1}^{n} u_i\{\mu_A^{\beta}(x_i)+(1-\mu_A(x_i))^{\beta}\}}\right)\right]^{\frac{1}{1-\alpha}}$$

Using the inequality (2.3), we get

$$\left[\sum_{i=1}^{n}\{\mu_A^{\beta}(x_i) + (1-\mu_A(x_i))^{\beta}\}\left(\frac{u_i}{\sum_{i=1}^{n} u_i\{\mu_A^{\beta}(x_i)+(1-\mu_A(x_i))^{\beta}\}}\right)^{\frac{1}{\alpha}} D^{-l_i\left(\frac{\alpha-1}{\alpha}\right)}\right]^{\frac{\alpha}{1-\alpha}} \ge$$

$$\left[\frac{\sum_{i=1}^{n} u_i\{\mu_A^{\beta}(x_i)\mu_B^{\alpha-1}(x_i)+(1-\mu_A(x_i))^{\beta}(1-\mu_B(x_i))^{\alpha-1}\}}{\sum_{i=1}^{n} u_i\{\mu_A^{\beta}(x_i)+(1-\mu_A(x_i))^{\beta}\}}\right]^{\frac{1}{1-\alpha}} \tag{2.8}$$

Taking $0 < \alpha < 1$, raising both sides of (2.8) to the power $\frac{1-\alpha}{\alpha}$, we get

$$\left[\sum_{i=1}^{n}\{\mu_A^{\beta}(x_i)+(1-\mu_A(x_i))^{\beta}\}\left(\frac{u_i}{\sum_{i=1}^{n}u_i\{\mu_A^{\beta}(x_i)+(1-\mu_A(x_i))^{\beta}\}}\right)^{\frac{1}{\alpha}}D^{-l_i\left(\frac{\alpha-1}{\alpha}\right)}\right]\geq$$

$$\left[\frac{\sum_{i=1}^{n}u_i\{\mu_A^{\beta}(x_i)\mu_B^{\alpha-1}(x_i)+(1-\mu_A(x_i))^{\beta}(1-\mu_B(x_i))^{\alpha-1}\}}{\sum_{i=1}^{n}u_i\{\mu_A^{\beta}(x_i)+(1-\mu_A(x_i))^{\beta}\}}\right]^{\frac{1}{\alpha}} \qquad (2.9)$$

As $\alpha<1, \frac{1}{\left(1-D^{\frac{\alpha-1}{\alpha}}\right)}>0$, multiply both sides equation (2.9) by $\frac{1}{\left(1-D^{\frac{\alpha-1}{\alpha}}\right)}>0$, we get

$$\frac{1}{\left(1-D^{\frac{\alpha-1}{\alpha}}\right)}\left[\sum_{i=1}^{n}\{\mu_A^{\beta}(x_i)+(1-\mu_A(x_i))^{\beta}\}\left(\frac{u_i}{\sum_{i=1}^{n}u_i\{\mu_A^{\beta}(x_i)+(1-\mu_A(x_i))^{\beta}\}}\right)^{\frac{1}{\alpha}}D^{-l_i\left(\frac{\alpha-1}{\alpha}\right)}\right]\geq$$

$$\frac{1}{\left(1-D^{\frac{\alpha-1}{\alpha}}\right)}\left[\frac{\sum_{i=1}^{n}u_i\{\mu_A^{\beta}(x_i)\mu_B^{\alpha-1}(x_i)+(1-\mu_A(x_i))^{\beta}(1-\mu_B(x_i))^{\alpha-1}\}}{\sum_{i=1}^{n}u_i\{\mu_A^{\beta}(x_i)+(1-\mu_A(x_i))^{\beta}\}}\right]^{\frac{1}{\alpha}}$$

Adding both sides by $\frac{1}{\left(D^{\frac{\alpha-1}{\alpha}}-1\right)}$ and after simplifying, we get

$$L_{\alpha}^{\beta}(A;U)\geq I_{\alpha}^{\beta}(A;B;U)$$

For $\alpha>1$, the proof follows along the similar lines.

Theorem 2.2:- For every code with lengths $l_1, l_2, \ldots, l_n$ satisfies the condition (2.3), L_{α}^{β} (A; U) can be made to satisfy the inequality,

$$L_{\alpha}^{\beta}(A;U)\geq I_{\alpha}^{\beta}(A;B;U)D^{\frac{1-\alpha}{\alpha}}+\frac{1-D^{\frac{1-\alpha}{\alpha}}}{1-D^{\frac{\alpha-1}{\alpha}}} \qquad (2.10)$$

Proof:- Let l_i be the positive integers satisfying the inequality

$$-\log\left(\frac{u_i(\mu_B^{\alpha}(x_i)+(1-\mu_B(x_i))^{\alpha})}{\sum_{i=1}^{n}u_i\{\mu_A^{\beta}(\mathbf{x_i})\mu_B^{\alpha-1}(\mathbf{x_i})+(1-\mu_A(\mathbf{x_i}))^{\beta}(1-\mu_B(\mathbf{x_i}))^{\alpha-1}\}}\right)\leq l_i<$$
$$-\log\left(\frac{u_i(\mu_B^{\alpha}(x_i)+(1-\mu_B(x_i))^{\alpha})}{\sum_{i=1}^{n}u_i\{\mu_A^{\beta}(\mathbf{x_i})\mu_B^{\alpha-1}(\mathbf{x_i})+(1-\mu_A(\mathbf{x_i}))^{\beta}(1-\mu_B(\mathbf{x_i}))^{\alpha-1}\}}\right)+1 \qquad (2.11)$$

Consider the interval

$$\delta_i=\left[\begin{array}{l}-\log\left(\frac{u_i(\mu_B^{\alpha}(x_i)+(1-\mu_B(x_i))^{\alpha})}{\sum_{i=1}^{n}u_i\{\mu_A^{\beta}(\mathbf{x_i})\mu_B^{\alpha-1}(\mathbf{x_i})+(1-\mu_A(\mathbf{x_i}))^{\beta}(1-\mu_B(\mathbf{x_i}))^{\alpha-1}\}}\right),\\ -\log\left(\frac{u_i(\mu_B^{\alpha}(x_i)+(1-\mu_B(x_i))^{\alpha})}{\sum_{i=1}^{n}u_i\{\mu_A^{\beta}(\mathbf{x_i})\mu_B^{\alpha-1}(\mathbf{x_i})+(1-\mu_A(\mathbf{x_i}))^{\beta}(1-\mu_B(\mathbf{x_i}))^{\alpha-1}\}}\right)+1\end{array}\right] \qquad (2.12)$$

Of length 1.In every δ_i , there lies exactly one positive integer l_i such that

$$0<-\log\left(\frac{u_i(\mu_B^{\alpha}(x_i)+(1-\mu_B(x_i))^{\alpha})}{\sum_{i=1}^{n}u_i\{\mu_A^{\beta}(\mathbf{x_i})\mu_B^{\alpha-1}(\mathbf{x_i})+(1-\mu_A(\mathbf{x_i}))^{\beta}(1-\mu_B(\mathbf{x_i}))^{\alpha-1}\}}\right)\leq l_i<$$
$$-\log\left(\frac{u_i(\mu_B^{\alpha}(x_i)+(1-\mu_B(x_i))^{\alpha})}{\sum_{i=1}^{n}u_i\{\mu_A^{\beta}(\mathbf{x_i})\mu_B^{\alpha-1}(\mathbf{x_i})+(1-\mu_A(\mathbf{x_i}))^{\beta}(1-\mu_B(\mathbf{x_i}))^{\alpha-1}\}}\right)+1 \qquad (2.13)$$

We will first show that the sequence $l_1, l_2, \ldots, l_n$ thus defined satisfies (2.3). From the left inequality of (2.13), we have

$$-\log\left(\frac{u_i(\mu_B^{\alpha}(x_i)+(1-\mu_B(x_i))^{\alpha})}{\sum_{i=1}^{n}u_i\{\mu_A^{\beta}(\mathbf{x_i})\mu_B^{\alpha-1}(\mathbf{x_i})+(1-\mu_A(\mathbf{x_i}))^{\beta}(1-\mu_B(\mathbf{x_i}))^{\alpha-1}\}}\right)\leq l_i$$

Or equivalently we can write

$$\left(\frac{u_i(\mu_B^\alpha(x_i)+(1-\mu_B(x_i))^\alpha)}{\sum_{i=1}^n u_i\{\mu_A^\beta(\mathbf{x_i})\mu_B^{\alpha-1}(\mathbf{x_i})+(1-\mu_A(\mathbf{x_i}))^\beta(1-\mu_B(\mathbf{x_i}))^{\alpha-1}\}}\right) \geq D^{-l_i} \tag{2.14}$$

Multiply both sides equation (2.14) by

$$\left(\mu_A^\beta(\mathbf{x_i})\mu_B^{-1}(\mathbf{x_i}) + (1-\mu_A(\mathbf{x_i}))^\beta(1-\mu_B(\mathbf{x_i}))^{-1}\right)$$

And summing over $i = 1,2,3,\dots,n$ on both sides to the resultant expression we get (2.3). The last inequality of (2.13) gives

$$l_i < -\log\left(\frac{u_i(\mu_B^\alpha(x_i)+(1-\mu_B(x_i))^\alpha)}{\sum_{i=1}^n u_i\{\mu_A^\beta(\mathbf{x_i})\mu_B^{\alpha-1}(\mathbf{x_i})+(1-\mu_A(\mathbf{x_i}))^\beta(1-\mu_B(\mathbf{x_i}))^{\alpha-1}\}}\right)+1$$

Or equivalently we can write

$$D^{l_i} < \left(\frac{u_i(\mu_B^\alpha(x_i)+(1-\mu_B(x_i))^\alpha)}{\sum_{i=1}^n u_i\{\mu_A^\beta(\mathbf{x_i})\mu_B^{\alpha-1}(\mathbf{x_i})+(1-\mu_A(\mathbf{x_i}))^\beta(1-\mu_B(\mathbf{x_i}))^{\alpha-1}\}}\right)^{-1} D$$

For $0 < \alpha < 1$, raising both sides to the power $\frac{1-\alpha}{\alpha} > 0$, we get

$$D^{-l_i\left(\frac{\alpha-1}{\alpha}\right)} < \left(\frac{u_i(\mu_B^\alpha(x_i)+(1-\mu_B(x_i))^\alpha)}{\sum_{i=1}^n u_i\{\mu_A^\beta(\mathbf{x_i})\mu_B^{\alpha-1}(\mathbf{x_i})+(1-\mu_A(\mathbf{x_i}))^\beta(1-\mu_B(\mathbf{x_i}))^{\alpha-1}\}}\right)^{\frac{\alpha-1}{\alpha}} D^{\frac{1-\alpha}{\alpha}} \tag{2.15}$$

Multiply equation (2.15) both sides by

$$\mu_A^B(x_i) + (1-\mu_A(x_i))^B\left(\frac{u_i}{\sum_{i=1}^n u_i\{\mu_A^\beta(x_i)+(1-\mu_A(x_i))^\beta\}}\right)^{\frac{1}{\alpha}}$$

And then summing over $i = 1,2,3,\dots,n$, both sides to the resultant expression we get,

$$\left[\sum_{i=1}^n\{\mu_A^\beta(x_i)+(1-\mu_A(x_i))^\beta\}\left(\frac{u_i}{\sum_{i=1}^n u_i\{\mu_A^\beta(x_i)+(1-\mu_A(x_i))^\beta\}}\right)^{\frac{1}{\alpha}} D^{-l_i\left(\frac{\alpha-1}{\alpha}\right)}\right] <$$

$$\left[\frac{\sum_{i=1}^n u_i\{\mu_A^\beta(x_i)\mu_B^{\alpha-1}(x_i)+(1-\mu_A(x_i))^\beta(1-\mu_B(x_i))^{\alpha-1}\}}{\sum_{i=1}^n u_i\{\mu_A^\beta(x_i)+(1-\mu_A(x_i))^\beta\}}\right]^{\frac{1}{\alpha}} D^{\frac{1-\alpha}{\alpha}} \tag{2.16}$$

Multiply both sides equation (2.16) by $\frac{1}{\left(1-D^{\frac{\alpha-1}{\alpha}}\right)} > 0$ then adding both sides by $\frac{1}{\left(D^{\frac{\alpha-1}{\alpha}}-1\right)}$ and after suitable operations, we get

$$L_\alpha^\beta(A;U) \geq I_\alpha^\beta(A;B;U)D^{\frac{1-\alpha}{\alpha}} + \frac{1-D^{\frac{1-\alpha}{\alpha}}}{1-D^{\frac{\alpha-1}{\alpha}}}$$

For $\alpha > 1$, the proof follows along the similar lines,

REFERENCES

1. Zadeh, L.A. [1965]: Fuzzy sets, Information and control, Vol. 8, pp. 338-353
2. Belis, M. and Guiasu, S. [1968]: A quantitative and qualitative measure information in cybernetic System, IEEE Transaction on information theory, Vol.IT-14, pp. 593-594
3. Longo, G. [1972]: Quantitative-Qualitative Measures of Information, Springer-Verlag, New York.
4. Taneja, H.C and Tuteja, R.K. [1985]: Characterization of Quantitative-Qualititative measure of inaccuracy, Kybernetika, Vol. 22, pp. 393-402
5. Guiasu, S and Picard, C.F. [1971]: Borne inferieture dela Longuerur utile de certainscodes, C.R Acad.Sci., Paris, Vol. 273, pp.248-251
6. Kraft, L.J. [1949]: A device for quantizing grouping and coding amplitude modulates pulses, M. S. Thesis, Department of Electrical Engineering, MIT, Cambridge
7. Autar, R and Khan, A.B. [1989]: On generalized useful information for incomplete distribution, Journal of combinatorics, Information and System Sciences, Vol. 14(4), pp. 187-191
8. Kerridge, D.F. [1961]: Inaccuracy and inference, Journal of Royal Statistical Society S. Vol. 23, pp. 184-194

9. De Luca, A and Termini, S. [1972]: A Definition of Non-probabilistic Entropy in the Setting of fuzzy sets theory, Information and Control, Vol.20, pp. 301-312
10. Shannon, C. E. [1948]: A mathematical theory of communication, Bell System Technical Journal, Vol.27, pp.379-423, 623-659
11. Baig, M.A.K and Javid, M. [2014]: Fuzzy coding theorems on generalized fuzzy cost measure, Asian Journal of Fuzzy and Applied Mathematics (ISSN: 2321-564X), Vol. 02, Issue, 01.
12. Baig, M.A.K and Javid, M. [2014]: Some new results on fuzzy directed divergence measures and Their inequalities, Asian Journal of Mathematics and Statistics, Vol. 7, pp. 12-20
13. Baig, M.A.K and Javid, M. [2014]: Some new generalization of fuzzy average code-word length and their Bounds, American Journal of Applied Mathematics and Statistics. Vol. 2, pp.73-76
14. Baig, M.A.K and Javid, M. [2014]: Some coding theorems on fuzzy entropy function depending upon Parameter R and V, IOSR Journal of Mathematics, Vol. 9, pp. 119-123
15. Baig, M.A.K and Javid, M. [2013]: Some generalization of fuzzy average length and inequalities, International Journal of Statistics and Analysis. Vol. 3, pp. 393-400
16. Baig, M.A.K and Rayees. [2006]:A generalized useful inaccuracy of order α and type β and coding Theorems, International Journal of Pure and Applied Mathematics, Vol. 32(4), pp. 467-474
17. Baig, M.A.K and Rayees. [2007]: Some noiseless coding theorems of inaccuracy measure of order α and Type β, Sarajevo Journal of Mathematics, Vol. 3(1), pp. 137-143
18. Hooda, D. S. [2004]: On generalized measures of fuzzy entropy, Mathematica Slovaca, Vol. 54, pp.315-325.